CURRICULUM, PEDAGOGY AND BEYOND

THE MATHEMATICAL ASSOCIATION OF VICTORIA

MAV24 CONFERENCE

MAV 2024 Conference - Mixing it up while keeping things in proportion

Years 7 – 10

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Today's session

Many practical problems in everyday life, including finance, measurement, science, and health are ratio and proportion problems. Solving proportion problems involves two simple arithmetic operations of multiplication and division, once the problem has been suitably formulated.

This session uses examples to show how to set up and solve proportion problems, covering set to set (part to part) and subset to set (part to whole) ratios and applying the unitary (divide and multiply) method using a simple template.

Session outline

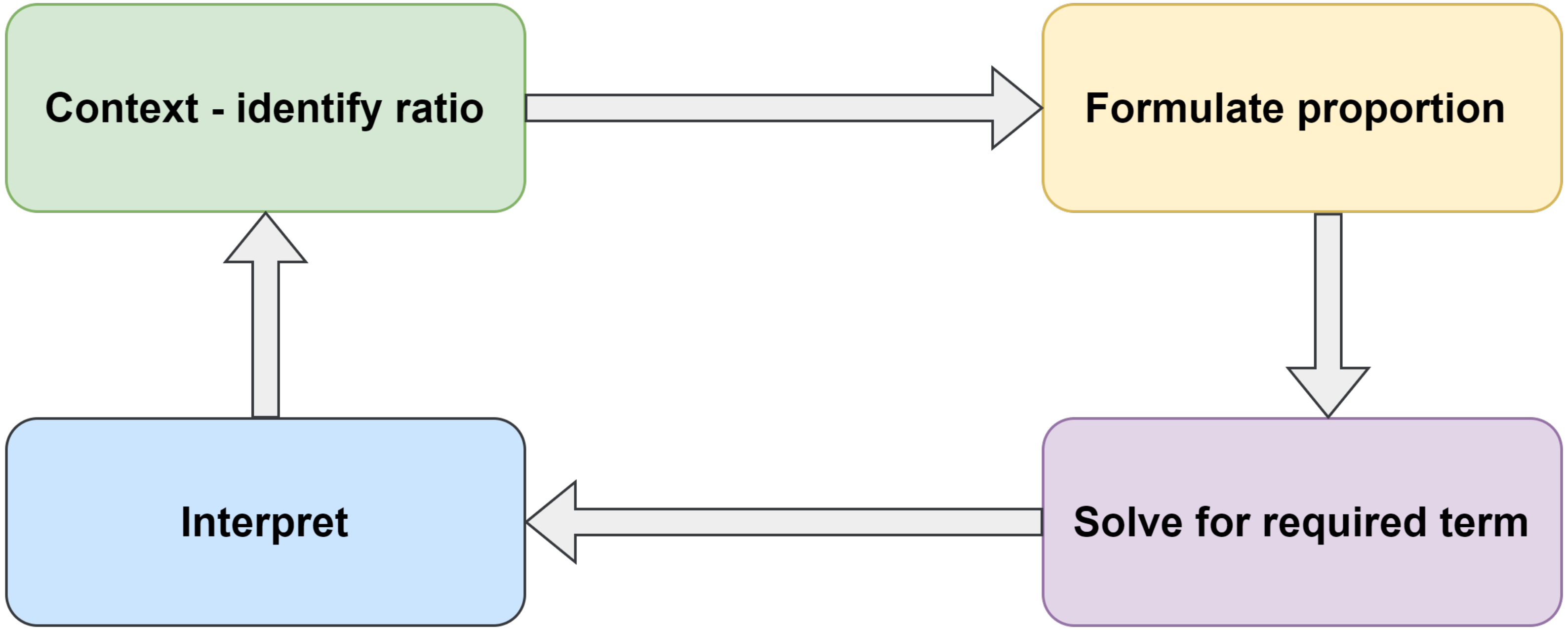
1. Activity (mixing it up while keeping things in proportion)
2. Overview (proportion problem-solving schema, key concepts of ratio, proportion, rates and percentages and methods for finding proportions)
3. Curriculum mapping (VC Version 2.0)
4. Resource links
5. Questions?
6. Thank you

Overview

Key ideas of ratio and proportion

- a ratio is a comparison of two or more quantities or magnitudes of a like kind (for example, the aspect ratio for a computer screen compares length with length)
- ratio can be constructed as part : part (set : set) or part : whole (subset : set)
- a rate is a comparison of quantities or magnitudes of different kinds (for example density compares mass with volume)
- a proportion is an equivalent ratio or rate
- a percentage is a ratio, rate or proportion where one of the terms is 100
- proportion is a multiplicative relation, solving proportion problems involves multiplication and/or division
- direct proportion between two variables y and x is represented by a linear relation $y/x = k$ or $y = kx$, where k is a non-zero real number, called the constant of proportionality, and the corresponding graph passes through the origin (0, 0), for example $\textit{circumference} = \pi \times \textit{diameter} \approx 3.14 \times \textit{diameter}$

A proportion problem-solving schema



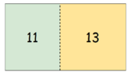
Ratio

Ratios, rates, and proportions are typically expressed in context, for example a ratio can be interpreted as a part-part (set : set) statement or a part-whole (subset : set) statement, using a '… to … ' format.

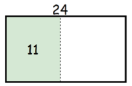
If a mathematics class has 11 students who own a pet and 13 students who don't own a pet, then the ratio of pet-owning students to non-pet-owning students is 11 : 13. This is a part-part (set : set) ratio.
In total the class has 24 students, and an alternative way of writing the previous ratio is as the ratio of pet-owning students to total number of students, 11 : 24 and similarly for the ratio of non-pet-owning students to total number of students, 13 : 24. These are part-whole (subset: set) ratios.

In any given context, both part-part (set : set) and part-whole (subset : set) ratios can be used to formulate and solve problems as applicable. In the mathematics class example, there are three ratios that can be expressed for a two-term ratio:

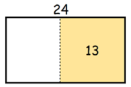
- pet-owning students : non-pet-owning students = 11 : 13



- pet-owning students : total students = 11 : 24



- non-pet-owning students : total students = 13 : 24



Rates

Rates and proportions are also typically expressed in context, rates are like ratios except they compare quantities or magnitudes of different kinds. Rates are usually expressed using a '... per ... ' format with the corresponding units written as '... / ...' or as a percentage.

Some common examples of rates are:

- average speed in kilometres per hour, such as 85 km/h (comparing distance travelled to time taken)
- heart rate in beats per minute, such as 72 bpm (comparing number of heartbeats to time)
- density in kilograms per cubic metre, such as air is 1200 kg/cubic metre (comparing mass to volume)
- best buys (comparing mass or volume to cost, such as $1.10/litre)
- tax, inflation (currently 3 – 5 % range), depreciation or other rates (income to tax, current to previous) – these are expressed as percentages
- currency conversion and exchange rates (27 Nov 2024 AUD = 0.6476US, Exchange Rates | RBA)

Percentage (%)

A percentage is simply a ratio or rate where one of the terms is 100 (from the Latin per centum). This can involve part-part (set : set) statement or a part-whole (subset : set) statement.

- In the mathematics class with 11 male students and 13 female students, the percentage of male and female students are found by solving the proportions 11 : 24 = ? : 100 and 13 : 24 = ? : 100 respectively
- If a person makes $ 25 000 in sales one month, and $ 37 000 in sales the following month, the percentage increase in sales is found by solving the proportion 25 0000 : 37 000 = 100 : ?

There are three basic percentage problems (and variations of these):

- find a given percentage of something (e.g., find 35% of 127, so 35 : 100 = ? : 127)
- find what percentage one thing is of another thing (e.g., what percentage of 67 is 94, so 94 : 67 = ? : 100)
- find the thing that is a given percentage of another thing (e.g., 54% of what number is 19? so 54: 100 = 19 : ?)

Basic Problems on Percentage | Real-life Problems on Percentage (math-only-math.com)

In proportion or not?

Finding proportions involves multiplication and/or division of both terms in a ratio by the same quantity, and various schema for this have been used over time. Effectively these set up an initial ratio and then compare with a corresponding ratio for which only one of the terms is specified.

That is, given $a : b$ what is $c : ?$ or what is $? : d$.

This is not an additive relationship.

	2	:	3
+	1	:	1
=	3	:	4

✘ not in proportion

	2	:	3
+	2	:	2
=	4	:	5

✘ not in proportion

	2	:	3
	2	:	3
=	4	:	6

✓ in proportion (doubles both)

Initially students encounter cases based on a simple integer multiple and/or a simple integer divisor, followed by cases which involve simple fraction and then decimal terms.

Problem solving schema

For many practical problems involving proportions of measured quantities and/or costs, there may be several terms involved in specifying a ratio or rate, and these terms may have decimal values, for example, quantities of concrete by mass or volume specified initially by a ratio of:

water : cement : sand : gravel = 1 : 2 : 4 : 6

Note that while the ratio of component quantities is a part : part (set : set) ratio, the practical problem involves knowing how much concrete is required, and part : whole (subset : set) ratios.

An important consideration is the use of proportional reasoning across the curriculum and the range of applications with contextually realistic data values. Given a particular context, practical modelling and problem-solving involves three key stages:

- Identification of relevant ratio or rate and formulation of required proportion
- selection of appropriate solution/computational strategy and implementation
- interpretation

The basic proportion problem

- A proportion expresses an equivalence between two ratios $a : b = c : d$.

- This means that there is a correspondence between a and c and the same correspondence between b and d where there is a non-zero real number k such that $k \times a = c$ and $k \times b = d$.

- The number k is called the constant of proportionality.

- The proportion can also be expressed using fractions as $a/b = c/d$ or $b/a = d/c$

- If $a : b$ is a given ratio, then the basic proportion problem involves finding one of c or d given the other.

a	:	b
↓		↓
c	:	?

a	:	b
↓		↓
?	:	d

The basic proportion problem – unitary method

	a	:	b	
$\div a$	$\downarrow$		$\downarrow$	
	1	:	$\frac{b}{a}$	
$\times c$	$\downarrow$		$\downarrow$	
	c	:	$? = \frac{b}{a} \times c$	

Have a go, solve 17 : 29 = 26 : ?

	17	:	29	
$\div 17$	$\downarrow$		$\downarrow$	$\div 17$
	1	:		
$\times 26$	$\downarrow$		$\downarrow$	$\times 26$
	26	:		

Solution – unitary method

Solve 17 : 29 = 26 : ?

	17	:	29	
÷ 17	↓		↓	÷ 17
	1	:	1.706	
× 26	↓		↓	× 26
	26	:	44.35	

Using fractional form

$$a : b = c : ?$$

$$\frac{b}{a} = \frac{?}{c}$$

$$\frac{b}{a} \times c = ?$$

$$4 : 11 = 13 : ?$$

$$\frac{11}{4} = \frac{?}{13}$$

$$\frac{11}{4} \times 13 = ?$$

$$\frac{143}{4} = ?$$

$$1.47 : 5.32 = ? : 6.83$$

$$\frac{1.47}{5.32} = \frac{?}{6.83}$$

$$\frac{1.47}{5.32} \times 6.83 = ?$$

$$? = 1.89 \text{ (2dp)}$$

Have a go, solve: 0.851 : ? = 19.043 : 2.510

Solution – fractional form

Solve: 0.851 : ? = 19.043 : 2.510

$$0.851 : ? = 19.043 : 2.510$$

$$\frac{?}{0.851} = \frac{2.510}{19.043}$$

$$? = 0.851 \times \frac{2.510}{19.043}$$

$$? = 0.112 \text{ (3 decimal places)}$$

Cross multiplication (product)

- This is a method based on the notion that for equivalent fractions, the cross products of numerators with denominators are equal.

  Let $\frac{a}{b} = \frac{c}{d}$, multiply both fractions by bd to get $\frac{a}{b} \times bd = \frac{c}{d} \times bd$, then simplify to $ad = bc$.

- Essentially, for a proportion $a : b = c : d$ it says ' the product of first and last terms is equal to the product of the middle terms'.

- It leads to a simple equation, into which the three known terms can be substituted, a product calculated and then the unknown term determined by dividing this product by the other known term.

- This provides a straightforward computational approach, however understanding which terms from the ratios relate to what part of the formulation is important, as these have been 'mixed' across the fractions. For example, to solve $11 : 3 = x : 7$, this is the same as solving $77 = 3x$, then $x = 77 \div 3$.

- Have a go, solve $3.14 : 5 = c : 11.1$

Solution – cross multiplication (product)

Solve 3.14 : 5 = *c* : 11.1

$$5c = 3.14 \times 11.1$$
$$5c = 34.854$$
$$c = 34.854 \div 5$$
$$c = 6.971 \text{ (3 decimal places)}$$

The rule of three (a bit of history)

Schemas that illustrate the unitary method can be used to explain what the constant of proportionality is for a given problem, leading to some version of what is called 'the rule of three' from the 17th Century CE, for example as explained in a popular arithmetic book of the time Cocker's *Arithmetick (Being a Plain and Familiar Method Suitable to the Meanest Capacity for the Full Understanding of That Incomparable Art*):

"Again, observe, that of the three given numbers, those two that are of the same kind, one of them must be the first, and the other the third, and that which is of the same kind with the number sought, must be the second number in the rule of three; and that you may know which of the said numbers to make your first, and which your third, know this, that to one of those two numbers there is always affixed a demand, and that number upon which the demand lieth must always be reckoned the third number."

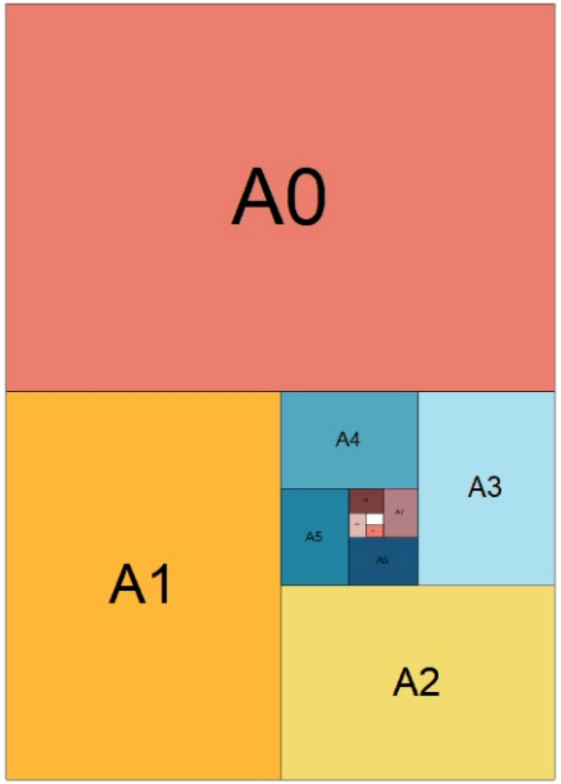
Clearly, problems involving ratio and proportion have been providing challenges in school education for some time!

Ratio and proportion in school mathematics

- experimental probability as long run proportion
- proportions and percentages in statistics and sampling
- financial mathematics
- constant difference/rate of change of a linear function, gradient of the graph of a linear function
- constant ratio/percentage rate of change for an exponential function
- similarity of shapes and objects, dilation transformations (enlargements and reductions)
- rational and irrational numbers
- trigonometric ratios (sin, cos, tan)
- percentage error in measurement: (measured – actual) / actual ˆ 100
- direct and inverse variation

Ratios that correspond to irrational numbers

- pi, or π, is the ratio of the circumference of a circle to its diameter, that is $\pi = C/d$
- π is an irrational number, that is, it can not be expressed as the ratio of two natural numbers
- rational approximations for π are 3 (very rough but useful for mental calculations), 3.14, 22/7 and 355/113
- The square root of 2 or $\sqrt{2}$ is the ratio of the length of the diagonal of a square to its side, a rough approximation is 1.4 = 7/5 and 99/70
- This ratio is the basis of the ISO paper size series
- A4 paper format / International standard paper sizes (cam.ac.uk)
- Going beyond the Golden Ratio. | Extreme Learning



Source: Going beyond the Golden Ratio. | Extreme Learning

Ratios, rates and proportions in other studies

- Art and design: similarity and scale, aesthetic proportions
- Geography: slope and scale
- Finance and economics: price comparisons, currency conversion, inflation, tax rates and scales, discounts, price increases, price-earnings
- Chemistry: molecular formulas, ratios in compounds
- Health and physical education: heart and exercise rates, food composition and nutrition
- Recipe with ingredients to scale per person:
  [Slow Cooked Lamb Shanks in Red Wine Sauce | RecipeTin Eats]

Activity

Mixing it up while keeping things in proportion

A concrete example

Task: design a concrete path for a house, calculate the volume of concrete required to lay the path, and the cost of materials.

For example, a 10 m long path, 1.5 m wide and 0.15 m deep.

Choose a concrete mix ratio for cement : sand : gravel (for example 1 : 2 : 3 = 2 : 4 : 6) and determine what quantities of these will be required.

Find out costs for each material (this will be determined by a *rate*, for example $/cubic metre or $ /kg).

Work out how much water will be used, for example 15 – 20% by volume, or a ratio of

water : cement : sand : gravel = 1 : 2 : 4 : 6

Specify the amounts of materials required and the total cost.

Some materials costs (these can vary)
1 m^3 of sand ~ $100
1 m^3 of gravel ~ $90
A 20kg bag of cement ~0.01m^3 ~$8
1 m^3 of water = 1000 L ~ $3

Solution – a concrete example

Volume of concrete required = 10 × 1.5 ˆ 0.15 = 2.25 m^3

Ratio of components by volume is 1 : 2 : 4 : 6 which is 13 parts in total.

Divide the volume 2.25 m^3 by the number of parts, 13, to get 2.25 ÷ 13 = 0.173 m^3 per part.

Multiply each component by required number of parts and cost/m^3 and round to a reasonable estimate

Water 1 ˆ 0.173 ˆ \$3 ≈ \$1

Cement (2 ˆ 0.173 /0.01) ˆ \$8 ≈ \$280

Sand 4 ˆ 0.173 ˆ \$100 ≈ \$70

Gravel 6 ˆ 0.173 ˆ \$90 ≈ \$100

TOTAL ~ \$ 450

Some more concrete information

- The cost of 1m^3 = 1000 L of pre-mixed concrete is around $ 200 – 300 + delivery and waiting costs
- 1 m^3 of concrete weighs ~ 2 400 kg or 2.4 metric ton
- 1 m^3 of concrete can be made up from 150L of water (1L water ~1kg weight), 250kg of cement, 700kg of sand and 1200 kg of gravel
- 20kg bag of premix concrete ~ $8

- What are the Proper Concrete Mix Proportions? (bnproducts.com)
- Cement Ratio Mixing Guide [2022] - Specifier Australia
- What are the Correct Concrete Mixing Ratios - Ratio Chart (everything-about-concrete.com)
- Cement & Concrete Mix Ratio - Aus STD Guide (buildsearch.com.au)
- Concrete Mix Ratio | What Is Concrete Mix Ratio | Types of Concrete Mix Ratio (civiljungle.com)
- How To Lay A Concrete Path - Australian Handyman Magazine
- CCAA_DATASHEET_Residential_driveways_and_paths_TT_Review1.pdf
- 2022 Concreting Cost Guide- Find concreters | iseekplant | iseekplant

Roman concrete

Why was Roman concrete so good?

It's all in the mixture!
The Romans were the first civilization to use concrete extensively as a construction medium, from around 200BCE

Riddle solved: Why was Roman concrete so durable? | MIT News | Massachusetts Institute of Technology

2,050-year-old Roman tomb offersinsightsonancient concrete resilience | MIT News | Massachusetts Institute of Technology

https://images.pexels.com/photos/29546110/pexels-photo-29546110/free-photo-of-ancient-roman-theater-ruins-in-volterra-italy.jpeg?auto=compress&cs=tinysrgb&w=800

Curriculum mapping

Activity and AC: Mathematics V8.4 content descriptions

Mapping the activity to content descriptions

Year 7

- find equivalent representations of rational numbers and represent positive and negative rational numbers and mixed numbers on a number line VC2M7N03
- find percentages of quantities and express one quantity as a percentage of another, with and without digital tools VC2M7N07
- recognise, represent and solve problems involving ratios VC2M7N09
- use mathematical modelling to solve practical problems involving rational numbers and percentages, including financial contexts such as ‘best buys’; formulate problems, choosing representations and efficient calculation strategies, designing algorithms and using digital tools as appropriate; interpret and communicate solutions in terms of the situation, justifying choices made about the representation VC2M7N10
- identify the sample space for single-stage experiments; assign probabilities to the possible outcomes and predict relative frequencies for related experiments VC2M7P01

Mapping the activity to content descriptions (ctd)

Year 8

- solve problems involving the use of percentages, including percentage increases and decreases and percentage error, with and without digital tools VC2M8N05
- use mathematical modelling to solve practical problems involving rational numbers and percentages, including financial contexts involving profit and loss; formulate problems, choosing efficient mental and written calculation strategies and using digital tools where appropriate; interpret and communicate solutions in terms of the context, reviewing the appropriateness of the model VC2M8N06
- recognise and use rates to solve problems involving the comparison of 2 related quantities of different units of measure VC2M8M05
- use mathematical modelling to solve practical problems involving ratios and rates, including distance-time problems for travel at a constant speed and financial contexts; formulate problems; interpret and communicate solutions in terms of the situation, reviewing the appropriateness of the model VC2M8M07

Mapping the activity to content descriptions(ctd)

Year 9

- solve spatial problems, applying angle properties, scale, similarity, ratio, Pythagoras' theorem and trigonometry in right-angled triangles VC2M9M03
- calculate and interpret absolute, relative and percentage errors in measurements VC2M9M04
- use mathematical modelling to solve practical problems involving direct proportion, rates, ratio and scale, including financial contexts; formulate the problems and interpret solutions in terms of the situation; evaluate the model and report methods and findings VC2M9M05
- recognise the constancy of the sine, cosine and tangent ratios for a given angle in right-angled triangles using properties of similarity VC2M9SP01
- apply the enlargement transformation to shapes and objects using dynamic geometry software as appropriate; identify and explain, using language of similarity, ratio and scale, aspects that remain the same and those that change VC2M9SP02

Resource links

Resource links

- rates_and_ratio (amsi.org.au)
- TIMES MODULE M35 - Proportion (amsi.org.au)
- Proportion.pdf (amsi.org.au)
- TIMES M2 Unitary Method 3 (amsi.org.au)
- Ratio, rates and proportions resources (vu.edu.au)
- Ratio, Proportion & Rates of Change (maths.org)
- NRICH topics: Fractions, Decimals, Percentages, Ratio and Proportion (maths.org)
- Clip 19 Summary clip for Ratio and Proportion.pdf (nctm.org)
- Understanding ratio and proportion - FUSE - Department of Education & Training
- Ratio and Proportion - FUSE - Department of Education & Training
- Ratios and proportions and how to solve them (Algebra 1, How to solve linear equations) – Mathplanet
- Inquiry Maths – Proportion
- Percentage Calculator
- Percentage Calculator (calculatorsoup.com)
- Cement Concrete Calculator | PCC Calculator | RCC Calculator | Free Estimate of Cement, Sand and Aggregate
- Concrete Calculator: Cement, Sand and Aggregate Dry Quanitities

Feedback & accessing a copy of this PPT

I would welcome your thoughts on today's session.

This QR code will take you to a very short feedback form.

If you'd like a copy of the PPT for this session, you can either:

- include your email address when completing the feedback form, or
- email me directly: david@leighlancasterconsulting.com.au

Then I'll share a link over the next few days.

2024 MAV H23 - Mixing it up while keeping things in proportion

Feedback MAV

Please also complete the MAV feedback survey within the conference app.

Any quick questions?

Thank you

Leigh-Lancaster Consulting

https://leighlancasterconsulting.com.au